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DIRECT NUMERICAL SIMULATION OF FULLY DEVELOPED FLOW NEAR A FLAT PLATE

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Abstract

A traditional local approach is applied for simulation of a statistically steady flow near a flat plate. A local flow domain is placed far from the leading edge of the plate. A coupled system of the Prandtl and the Navier-Stokes equations for disturbances is chosen as the governing system of equations. Fully spectral approach is used for space discretization. This helps to avoid prescribing inflow and outflow boundary conditions. Acceptable particular solutions are selected to describe physically adequate phenomenon. A new spectral method for integrating the Cauchy problem is adopted to study the flow evolution.

1. Introduction

The phenomenon of turbulence in lengthy flows involves a large range of space scales. This is the challenge for flow modeling based the full Navier-Stokes equations. The class of external quasi-homogeneous flows related to the boundary layer theory has a quasi-regular nature, it shows that lengthy disturbances carry small amounts of energy in thin layers. The main feature of the boundary layer flows is the existence of laminar regimes of motion that can be described on the basis of the Prandtl equations. Thin laminar flows are unstable and simulation of thin transitional and turbulent flows is usually based on the full Navier-Stokes equations (see, for instance, [8]). The regular approach, however, introduces restrictions on Reynolds numbers because of limitations on computer resources. To weaken the restriction one can use local modeling of quasi-homogeneous external flows. We employ SFM, a coupled system of Prandtl and Navier-Stokes equations for disturbances [5,7] for local modeling of the flow near a flat plate.

First theoretical investigations of stability of laminar boundary layer flows showed the existence of coherent structures in transitional zones of the flows in the form of the Tollmien-Schlichting wave. The existence of local flow structures was confirmed later by experiments for both transitional and turbulent

quasi-regular flows. The local approach is widely used in the thin layer theory. Being the analogy with a channel flow, the approach leads to definition of a local Reynolds number, but it does not answer the question about the scale of flow structure. It also introduces an uncertainty to inflow and outflow boundary conditions because of the weak non-homogeneity of quasi-regular flows. The local approach is not general enough to follow space upwind peculiarities of a flow, but this can be considered as the advantage when simulating lengthy flows for high Reynolds numbers.

The scale of a flow structure is the result of competition of disturbances with random lengths that depend weakly on the external conditions and are determined by the internal nature of a flow, in particular, by the local Reynolds number. Space parameters of coherent structure can be estimated on the basis of numerical experiments. If a flow is unstable to infinitesimal disturbances then there is another possibility to choose the local characteristic size of the flow: one can suppose that scales of coherent flow structures coincide with space scales of the disturbances corresponding to the maximal temporal increments. We apply this *principle of maximal instability* for the near-wall flow modeling.

Turbulence has a self-sustained nature, it arises in flows spontaneously, and its physical existence does not require special conditions. This attribute of turbulence could be considered as the concept for numerical simulation. Actually, a laminar flow can loose its stability due to introduced disturbances at any place of the flow at some moment of time. The disturbances then develop in space and time, and the development does not require any conditions for its support, in particular, inflow boundary conditions. Application of spectral methods is the direct way of realizing this approach numerically. Such an approach allows developing the idea of selection of local particular solutions [3], which capture the adequate phenomenon when extracting them from the problem considered.

Another feature of the numerical procedure applied is the application of a new integration method for the Cauchy problem [2]. The algorithm is the adaptation of the general spectral approach to the Cauchy problem solving. Unknown variables are represented as a sum of preliminary chosen sequence of functions; both the spectral space and the space of original variables are involved in the algorithm construction. The approach leads to the choice of non-uniformly disposed collocation points in a fixed temporal interval and provides an additional flexibility in comparison to traditional methods.

2. Space and Time Discretization Procedures

To this end we developed the results for 2D flow modeling, so the discretization procedure is briefly described in two space dimensions and time.

Let (t, X_0, y) be a non-dimensional time and non-dimensional coordinates with the origin at the leading edge of the plate, and let the local variable x be connected to X_0 by the formulas

$$X_0 = 1 + X, \quad X = \lambda x, \quad \lambda = \kappa^2 / \mathrm{Re}, \quad \kappa = 1.72078766,$$

where Reynolds number Re is chosen on the basis of thickness of the boundary layer. The neutral stability curve $\mathrm{Re} = \mathrm{Re}(\alpha)$ borders energy-bearing intervals of space scales that support the phenomenon of turbulence in the flow. For a fixed Reynolds number, we can pick up the point (α_m, Re) inside the neutral curve that corresponds to the disturbance with the maximal temporal increment. In accordance with the principle of maximal instability, the parameter $\alpha = \alpha_m$ provides the scale of coherent structure $2\pi / \alpha$, and we study flow in the domain

$$D = \{-\pi / \alpha \le x \le \pi / \alpha \ , \ 0 \le y \le \infty\}.$$

We consider the solution of the problem for *disturbances* $\mathbf{F}^d$ of the laminar flow in the form

$$\mathbf{F}^d = \mathbf{F}^S + \mathbf{F}^f, \quad \overline{\mathbf{F}}^f = 0,$$

where $\mathbf{F}^S = \{u^S, v^S, p^S\}(X_0, y, t)$, $\mathbf{F}^f = \{u^f, v^f/\lambda, p^f\}(x, y, t)$ are slow and fast components of the velocity vector and pressure, bar means $x-$ averaging. The Navier-Stokes equations for disturbances $\mathbf{F}^d$ were split in coupled Prandtl and Navier-Stokes equations for the disturbance components $\mathbf{F}^S$ and $\mathbf{F}^f$ in [5,7]; the procedure was specified for smooth periodic function $\mathbf{F}^f$ of the x-variable in the domain D.

The problem statement does not involve unknown inflow and outflow boundary conditions, and we follow the idea of extracting the adequate particular solutions from the governing equations.

The approximation in x-variable is

$$\{u^f, v^f, p^f\}(x, y, t) = \sum_{\substack{|k| \le K \\ k \ne 0}} \{u_k^f, v_k^f, p_k^f\}(y, t) e^{i\alpha x}. \tag{1}$$

If weak non-homogeneity of the flow is taken into account in the equations for fast disturbances, then the requirement of periodicity of $\mathbf{F}^f$ leads to Gibbs phenomenon of the order of $O(1 / \mathrm{Re})$ near the inflow and the outflow boundary

for representation of slow components of the velocity vector. Modes in (1) draw the Orr-Sommerfield eigenfunctions in the development of disturbances.

The linearized Prandtl problem for disturbances also has nontrivial solutions in the absence of upwind boundary conditions. Those modes are proportional to the sequence $\{X_0^{-v_j}\}$, $v_1 = 1$, $v_2 = 1.887$, $v_3 = 2.867$, ... in the boundary layer coordinates [9]. It seemed the modes would have to be involved in the phenomenon modeling [7]. However, it has been shown that those eigenfunctions describe solely the peculiarities of damping the slow component of upwind stationary velocity disturbances in the streamwise direction, and they must be excluded from the description of the flow evolution in the case of uncertainty in the inflow boundary conditions [3]. The uncertainty makes the problem under consideration ill posed in terms of slow variables, and we apply the Galerkin-Petrov variant of a spectral method representing the solution in the longitudinal direction in the form

$$u^S = \sum_{j=0}^{J} U_j(\eta,t) X_0^j, \quad v^S = \sum_{j=0}^{J} V_j(\eta,t) X_0^{j-1/2}, \quad \eta = y/\sqrt{X_0} .$$

This removes the possibility of self-excitation of the slow modes. So, interaction of fast disturbances is the only source of excitation of slow ones. Variation of $X_0 = 1 + X$ is small if $\alpha \sim O(1)$, and we can choose functions X^j, $j = 1,...,J$ as the test functions being content with a small value of J, in particular, with $J = 1$, because terms of the order $O(\lambda^2)$ have been removed from the model equations.

The discretization procedure for the problem under consideration in the direction orthogonal to the wall (that is in y-direction) for both slow and fast components of the flow is based on a weak formulation (see, for instance, [4]). Orthogonal exponential polynomials $\mathcal{E}_{ml}(\alpha y)$, $l = 1,...,m$ [6] were used as the trial and the test functions.

The finite dimensional dynamic system of equations extracted from the governing equations was integrated in time by a spectral method developed recently in [2]. The method can be summarized as follows.

Let $\mathbf{Y}$ and $\mathbf{F}$ be vector functions with dimension N. We consider the Cauchy problem

$$\mathbf{Y}' = \mathbf{F}(t, \mathbf{Y}(t)), \quad \mathbf{Y}(T_I) = \mathbf{Y}_I \tag{2}$$

in the interval $t \in [T_I, T_I + h]$, $h > 0$, for some prescribed initial value $\mathbf{Y}_I$. Let $\lambda_m > 0$, $\alpha_n = h/\lambda_m$, $\tau = (t - T_I)/\alpha_n$, $\mathbf{y}(\tau) = \mathbf{Y}(T_I + \alpha_n \tau)$,

$\mathbf{f}(\tau, \mathbf{y}) = \alpha_n \mathbf{F}(T_I + \alpha_n \tau, \mathbf{Y}(T_I + \alpha_n \tau))$. Then problem (2) can be represented as follows

$$y'(\tau) = \mathbf{f}(\tau, \mathbf{y}(\tau)), \quad \mathbf{y}(0) = \mathbf{Y}_I, \ \tau \in [0, \lambda_m] \tag{3}$$

We look for an approximate solution $\mathbf{y}_n(\tau) \approx \mathbf{y}(\tau)$ to system (3) in the form

$$\mathbf{y}_n(\tau) = \mathbf{Y}_I + \mathbf{f}_0 \tau + \sum_{j=1}^{n} \mathbf{a}_{nj} (\mathcal{S}_{nj}(\beta_n, \tau) - (-1)^{n-j} \tau), \tag{4}$$

$$\mathcal{S}_{nj}(\beta_n, \tau) = \int_0^\tau \mathcal{E}_{nj}(\beta_n t) dt$$

that satisfies the initial condition in (3) and the initial condition for the derivative of the unknown variable. Here $\mathbf{a}_{nj}$ are unknown vector coefficients with dimension N, $\beta_n = 1$, $\mathbf{f}_0 = \mathbf{f}(0, \mathbf{Y}_I)$. To find the spectral coefficients $\mathbf{a}_{nj}$ in (4) we consider the discrete form of the problem (3) in the collocation points $\tau_{ns} = \lambda_{ns} / \beta_n$, $s = 1, \ldots, n$; λ_{ns} are the zeros of the polynomial $\mathcal{E}_{n0}(\tau)$, λ_m is the maximal zero. Transformations in the spectral space lead to the discrete analog of the Cauchy problem (2)

$$T_{np} = T_I + v_{np} h, \quad \mathbf{Y}_{np} = \mathbf{Y}_n(T_{np}),$$

$$\mathbf{Y}_{np} = \mathbf{Y}_I + \sigma_{np0} h \mathbf{F}(T_I, \mathbf{Y}_I) + h \sum_{s=1}^{n} \sigma_{nps} \mathbf{F}(T_{ns}, \mathbf{Y}_{ns}), \tag{5}$$

in the original variables. Here $p = 1, \ldots, n$, and coefficients $v_{np}, \sigma_{np0}, \sigma_{nps}$ can be tabulated for any reasonable n. Resolving the equations (5) leads to a step error of the order of $O(h^{n+1})$ for sufficiently smooth function $\mathbf{F}$, but getting the precise solution is a difficult task. An explicit single-step procedure that provides an approximate solution of the problem (5) has been developed as a trial value. The procedure is based on the property of thickening of the zeros of $\mathcal{E}_{n0}(\tau)$ in the vicinity of $\tau = 0$. This leads to a recurrence algorithm that has the same order as the Euler method, but gives better accuracy when increasing n. It should be mentioned that the algorithm is convenient for the implementation of the adaptive stepsize control.

Detailed description of the method is given in [2]. The method could provide calculations both in the spectral space and in the space of the original variables that gives the opportunity to interpolate unknown variables in

intermediate collocation points. High degree approximations lead to higher storage requirements, but calculations in highly non-uniform disposed points can give an advantage in terms of prediction of the solution and application to stiff problems.

The single-step explicit procedure has been applied for simulation of the flow for $n \leq 16$. The initial data at $t = 0$ should represent an arbitrary disturbance with small amplitude. Following the concept of self-sustained nature of turbulence, one can suppose that the disturbance contains the mode with the maximal temporal increment. So, we can chose the initial guess, considering only one mode at $t = 0$, and the initial data satisfying linearized governing equations can have a moderate amplitude.

3. Flow Simulation Results

Simulation has been performed for $N = 1220$ degrees of freedom and for large Reynolds numbers. Full development of disturbances was provided by integration in a long non-dimensional time interval. The example of the simulation described here corresponds to the turbulent zone of the flow for $Re_l = 10^6$, in this case the local Reynolds number is equal to 2961.1.
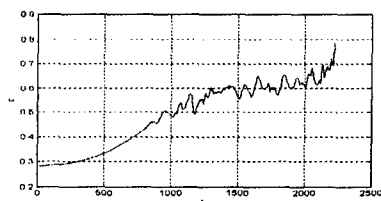

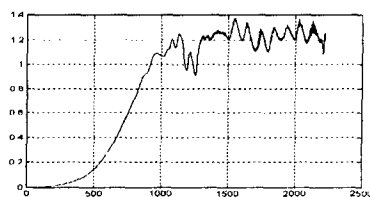*Figure 1*. Development of phase speed

Figure 2. Development of mean friction

Development of the phase speed c in time is shown in Fig. 1. The speed varies from the value that is determined by the linear theory of hydrodynamic stability until a value in the interval (0.6, 0.7). It is found that the phase speed of statistically steady flow is a rather conservative characteristic, it depends weakly on the local Reynolds number. Figure 2 shows the development of the slow component of the skin friction in the point $X_0 = 1$ on the wall. In contrast with the phase speed,
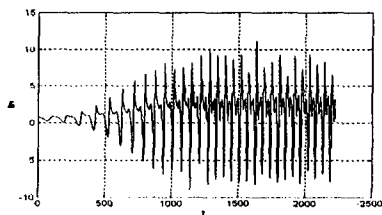

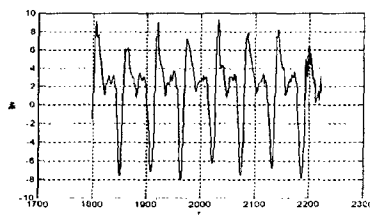*Figure 3*. Development of skin friction

Figure 4. Fully developed skin friction

mean skin friction increases significantly when increasing the distance from the leading edge. It appears that the model employed for the direct numerical simulation demonstrates good results in mean skin friction prediction. This may be the consequence of involving the Prandtl equations for disturbances to the flow description. Fig. 3 shows development of full skin friction on the wall, details of the statistically steady regime of the flow are given in Fig. 4, which is the part of Fig. 3.   It is seen that the impulsive injection of fluid into the boundary layer takes approximately 1/5 of the full time in every point of the domain near the wall. This 2D phenomenon correlates with the existence of hairpin vortex structure in 3D boundary layer [1].

Considering Fig. 5 and Fig. 6 one may come to the conclusion about chaotic nature of the flow. The power spectral density is shown in Fig. 5 in logarithmic scale. Two dimensional projection of the attractor of the dynamic system for fully developed flow in delay coordinates with delay $\Delta t = 30$ for full skin friction is shown in Fig. 6. The trajectory with self-intercepts clearly shows the complexity of the phenomenon and demonstrates the nature of the dynamic system extracted.
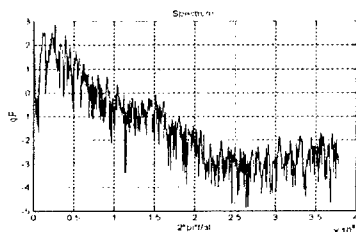

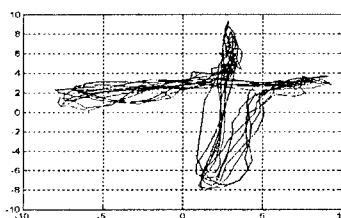
Figure 5. Power spectral density

Figure 6. Projection of the attractor in the delay coordinates

It should be mentioned that the dependence of the wall pressure on time is similar to the full skin friction; the spectral analysis shows that the characteristics of the pressure frequency correspond to the appropriate characteristics of the full skin friction, and the pressure phase speed is the same as for the skin friction.

4. Conclusion

The concept of local simulation of flows related to boundary layer problems is adopted in this work. Spectral methods based on selection of appropriate particular solutions are elaborated to overcome the uncertainty in inflow and outflow boundary conditions. The restrictive approach that leads to elimination of lengthy disturbances is used to choose the scale of a coherent structure. Special attention is paid to application of a recently developed spectral method for the initial value problem integration.

Although simulation of 2D flow for big Reynolds numbers can be considered as a methodical one, the results obtained are consistent with turbulence modeling. Heavy nonlinear phenomenon in the near-wall region has a chaotic background. Fluid is pumping away from the wall, and the region of separation moves along the wall taking approximately 1/5 of an exposed time in every point of the segment chosen on the wall. At the same time simulation shows good results for the mean profile of the flow. The phase speed of the flow is in the interval (0.6, 0.7). The parameters are found for $\mathrm{Re}_l = 10^6$.

5. Acknowledgments

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